

Name: _____ Date: _____

AP Calculus AB

Grade Boost

Ms. Etukudo

ALL PROBLEMS SHOULD BE DONE ON A SEPARATE SHEET TOTAL=71 PTS

1) Suppose that the line tangent to the graph of $f(x)$ at $x = 3$ passes through the points $(1, 2)$ and $(5, -4)$.

- [3 points] Find $f'(3)$.
- [3 points] Find $f(3)$.
- [3 points] Estimate the value of $f(2.9)$.

2) The popularity of baby names varies over time; the names that are popular one year may not be popular at all within a few years. The popularity of baby names beginning with the letter I appears to be periodic. In 1885, approximately 16,000 per million babies born had first names beginning with the letter I. Their popularity began decreasing at that time and decreased until 1945, when the number had dropped to a low of 2,200 per million. In 2005 it was back to 16,000 per million babies born. Let $B(t)$ denote the popularity of names beginning with the letter I, in thousands of babies per million babies born, t years after 1885. Assume that $B(t)$ is a sinusoidal function.

- [6 points] Sketch the graph of $B(t)$. (Remember to clearly label your graph.)

3) The H1N1 flu virus arrived in the US last spring. Data from the CDC for the region including Michigan, Minnesota, Illinois, Indiana, Wisconsin, and Ohio is shown in the table below. $H(t)$ denotes the cumulative number of cases of H1N1 flu in this region t weeks after August 15, 2009.

t	0	1	2	3	4	5	6
$H(t)$	8266	8314	8365	8482	8632	8903	9165

- [2 points] Evaluate and interpret $H(5)$.
- [3 points] Estimate $H'(5)$.

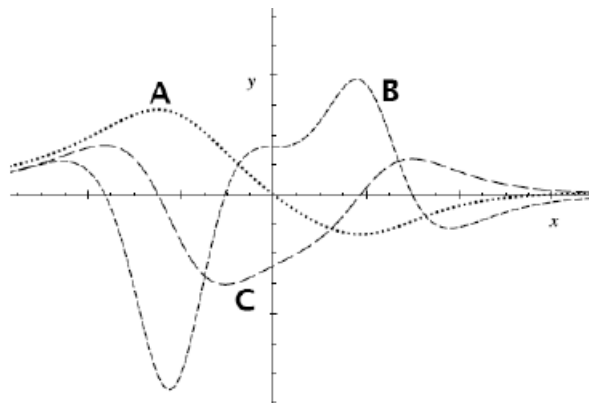
4) [16 points] Since it was first introduced, the number of users of the internet worldwide has increased dramatically. Let $I(t)$ denote the number (in millions) of worldwide internet users t years after 1995. Then $I(t)$ is given by the formula

$$I(t) = \begin{cases} 16(361/16)^{t/5} & \text{if } 0 \leq t \leq 5 \\ 361(1.18)^{t-5} & \text{if } 5 < t \leq 10 \\ A + 10(t - 10) & \text{if } t > 10 \end{cases}$$

- [3 points] Find A so that $I(t)$ is continuous.
- [4 points] Find the continuous growth rate of $I(t)$ in the year 1997.
- [3 points] Find the average rate of change of the number of internet users between 1995 and 2000.
- [6 points] Use the definition of the derivative to numerically estimate (i) $I'(7)$ and (ii) $I'(10)$.

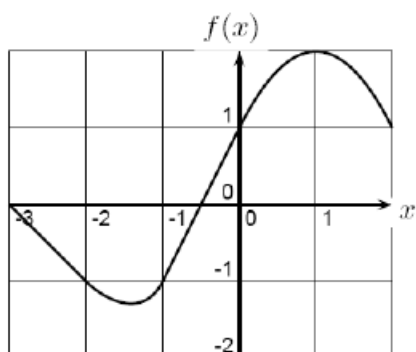
5) [6 points] Consider the function $W(t) = 3 \ln(\sin(t)^2 + 2)$. Write down the limit definition of $W'(t)$. (You do not need to estimate or compute the derivative.)

6) [9 points] The three graphs labeled A, B, and C below depict a function g along with its first and second derivatives (g' and g''). Determine which is which. Your answer to parts (a)-(c) should be a single legible capital letter (A, B, or C).



- [2 points] The graph of g is labeled _____.
- [2 points] The graph of g' is labeled _____.
- [2 points] The graph of g'' is labeled _____.
- [3 points] Briefly explain your reasoning.

7) (12 points) The graph of a function f is shown below, together with a table of values for its derivative f' . Let $g(x) = f(f(x))$.



x	$f'(x)$
-3	-1
-2	-1
-1	2
0	2
1	0
2	-2

- (2 points) Find $g(-2)$
- (3 points) Find $g'(-2)$
- (3 points) Write an expression for $g''(x)$ in terms of f and its derivatives.
- (4 points) Suppose $f''(-1) = 2$. What is $g''(-1)$?

8) (a) (4 points) Show that the point $(x, y) = (3, -6)$ lies on the curve defined by $y^2 - x^3 - x^2 = 0$.

(b) (4 points) What is the equation of the tangent line to the curve at the point $(3, -6)$?